Fields

Electrostatic forces are "Action at a Distance" – charges do not need to be in contact to push or pull on one another as we have seen in many demonstrations. Gravitational forces are also forces that act at a distance, and at first it seems odd to think that objects can apply forces without touching each other. One way people have developed to help get more comfortable with this idea is another concept: ***force fields***. (Michael Faraday invented this concept in the 1800's.)

The idea of a field is an extremely powerful one, but it can feel fairly unfamiliar at first***.*** You have probably heard the terms "electric field" or "magnetic field" but you may not have considered a precise definition of what these terms mean. Although fields might seem fairly abstract when you first encounter them, after a while physicists tend to think of the field as a very concrete part of nature, not just an abstraction. So before we continue, let's propose some definitions:

Scalar field: for every point (or region) of space we assign a number
Vector field: for every point (or region) of space we assign a vector

Let's begin with scalar fields. What is a quantity we could define for every point in space, say inside a room? One possibility would be temperature. How would we ***map*** this field? To assign a temperature to every point in the room we could walk around with a GPS, which would tell us the coordinates of the point, and for each point we could make a measurement with a thermometer. The screen on your computer or smartphone is also a scalar field in a sense. When you display a picture, for every pixel you can define a specific color, and you can imagine each color being associated with a number.

What about an example of a vector field? One example would be wind. You could imagine for every point in space defining a vector that represented the direction of the wind, as well as the speed of the wind. So for every point we would have a velocity vector for the wind. This field would be a bit trickier to map. We would need a GPS to tell us our position, a wind sock to tell us the direction of the wind, and an instrument to measure the wind speed (an anemometer). Or perhaps you would have a little sail, and by measuring the force on the sail you would know the wind's speed and direction.

Pilots use maps of wind fields when deciding on airplane routes. You can see examples of these maps in the link below (and if you click "wind streamlines" those will look very similar to the "electric field lines" we are going to discuss later):

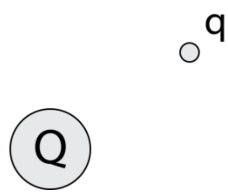
http://aviationweather.gov/adds/winds/

Hopefully, you will be able to connect some of the qualities of these example fields to the idea of the electric field.

Electric Fields

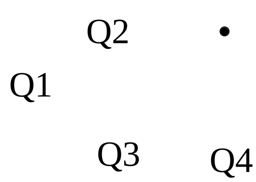
Now let's look at the idea of the electric field. A charge $+Q$ produces forces on any other charges, anywhere in the universe. Imagine putting a tiny positive charge $+q$ somewhere, a distance r away. Coulomb's law says the tiny charge $+q$ is repelled, and the force of $+Q$ on $+q$ is given by:

$$F_{Q\_on\_q} = \frac{kQq}{r^2}$$



Faraday argued there's an "electric force field" surrounding $+Q$. It's like a "state of readiness" to push any other charge that happens to come along. The field is not exactly physical; it is not something you can taste, or smell, or see. It just manifests itself if you put a charge (any charge) somewhere (anywhere!) This concept is strange but *very* useful. Stated another way, Faraday does not quite think about electric forces as "action at a distance": instead of thinking about Q pushing on q, far away, it is more like Q produces an electric force field everywhere, and it is that field *right wherever q is,* that finally pushes on q.

Suppose you have a bunch of charges. If you bring in one more little charge, q, anyplace you like, say the little black spot, it will feel an electric force in some direction. (You could figure it out by using Coulomb's law, adding the four separate forces on the little charge as vectors)



But again, you can instead think of an "electric force field" at the point, which tells you exactly which way a charge will be pushed if you put it there. The E-field is present *whether or not* you bother putting a charge there. It is present at any (and every) point in space.

In the analogy to a wind field, if you place a small sail at a point in a wind field it will feel a force. But the wind exists whether or not you put a sail there! In addition, if you put a large sail at the same point, it will feel a larger force. If you know the wind field at any point, you will be able to know the size and direction of the force on any sail you place at that point, once you know the size of the sail.

Let's first just define the electric field mathematically, and then justify and explain it more. We often refer to the electric field as the "E-field". The E-field should certainly be a vector field. It should also tell you the force on any charge q place at any particular point.

We define

$$\vec{E} = \frac{\vec{F}}{q}$$

Or, more carefully:

$$\vec{E}(at\ point\ p) = \frac{\vec{F}(on\ a\ charge\ q, at\ point\ p)}{q}$$

From this definition, the units of electric field will be Newtons/Coulomb, or N/C.

How would we map this field out? Physicists usually say that we map the electric field using a small ***positive*** "test" charge. By a "test" charge, we just mean a charge we are using to test the field. Why small? The reason people often say a "small" charge is because if we imagine mapping the field, we do not want disturb the charges producing the field, but the exact size of the charge should not matter. Why positive? That is just a convention; the direction of the field is going to tell us the direction a positive charge would feel a force (so a negative charge in the same place would feel a force in the opposite direction).

We could imagine hanging our small charge from a tiny spring, as we walk around, the charge will be pushed and pulled in different directions. At any point, the direction the charge feels a force will tell us the direction of the field, and the amount the spring stretches would measure the force on the charge at that point.

Why did we divide out the "test charge" q?

The reason is that the E-field is a property of space at any point. It shouldn't matter how much charge we used to test for it. If we bring in "q", I'll feel some force F. If I bring in "2q", Coulomb's law says I'll feel exactly twice the force, 2F. But since the E-field is "force divided by charge", in the second case (twice the force, twice the charge) the factors of two cancel, and comes out the exact *same* no matter what "q" is! That's what we want - E has some value at every point in space, whether or not there's any charge physically at that spot - and you can use it to *figure out* the force on *any* charge of any size that you bring to that spot.

Remember:

The electric field exists at a point whether or not there is a charge at that point!

Analogy to Motivate E fields:

Go to the grocery store and buy some sugar. On day one you buy 2 pounds, and pay \$4. On day 2 you buy 3 pounds, and pay \$6. What you pay depends on how much you buy. So, it might seem complicated to try to predict how much you'll have to pay another day, when you buy yet a different amount. But then you notice that

$$\frac{\$\, spent}{amount\ bough} = \frac{\$4}{2\ lbs} = \frac{\$6}{3\ lbs} = \frac{\$2}{lb}$$

There is a simple, underlying, universal, common ***unit price***.

$$\boldsymbol{unit\ price} = \$2/lb$$

So now you immediately know how much you'll pay, no matter how much you buy:

$$Price = \left(\frac{\$2}{lb}\right)(amount\ you\ buy)$$

Similarly with E-fields: E is like the "unit price per pound" (only here it's really "unit force per charge"). The force on a charge q seems complicated at first: different if you put in different charges. But then you notice that if you multiply both sides of the equation defining the E-field by the charge q you get:

$$Force = (charge)(Electric\ Field)$$

Knowing the E-field you can easily figure out the force on ***any*** charge now! That is one reason why the E-field is useful; it is like knowing the "unit price" at the store. If you know what the E-field is at any point in space, you can immediately figure out the force on ***any*** charge" placed at that point. Written more compactly than above, we have

$$\vec{F} = q\vec{E}$$

Another reason for the usefulness of the E-field is that once we have mapped it out, we can avoid an immense amount of work. Imagine there are 1 billion charges glued all around a room. Some positive, some negative, and all have different sizes, some big, some small. Suppose you now place a new charge somewhere in the room and want to know the force on this charge. One approach would be to use Coulomb’s law to calculate *1 billion different force vectors*, and then do vector addition on all those vectors to find the net force. Of course, if we move the charge to a new position, we would have to repeat this process all over again!

However, if we have mapped out the E-field, at any point we can compute the size and direction of the force on any charge just by multiplying the E-field vector by the size of the charge.

Notice: a negative charge feels a force in the opposite direction of the E-field vector

Even more conveniently, if we know an ***equation*** for the electric field formed by a particular arrangement of charge, we do not even need to map out the field; we can just use the equation!

Electric Field of a Single Point Charge

So what is the simplest "arrangement" of charge we can think of, and can we compute the equation for this arrangement? Computing the E-field is pretty much the same as finding the force, just remember to divide out q. So, if you have a single charge Q somewhere, and want to know the **E** field at some other point, just imagine putting a charge "q" at that point, and use Coulomb's law:

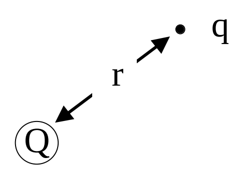
$$F_{Q\_on\_q} = \frac{kQq}{r^2}$$

$$\vec{E} = \frac{\vec{F}}{q}$$

So if we divide each side of the force equation by the test charge, the test charge cancels out and we have that

$$\vec{E}_{point\ charge} = \frac{kQ}{r^2}$$

(For a single charge Q, this tells |E|, a distance r away)



The direction of **E** is *radially outwards*, away from Q, if Q>0.
The direction of **E** is *radially inwards*, towards Q, if Q<0.
(That's just the statement that like charges repel, opposites attract)

Since **E** = **F**/q, the direction of **E** is the same as the direction of the force on a *positive* test charge.

E is defined at all points in space, and depends ONLY on Q, not on the test "q". Regions of space have a "possibility" of providing electric forces, even if no test "q" happens to be there at the moment. Similarly, gravitational fields exist at all points. There's a "g-field" in this room. It's downwards everywhere. If you DO release a pebble at some spot, the pebble immediately starts to accelerate down because of the "g-field". But the g-field is still there whether or not you put a "test pebble" at some spot to check.

By the way, you can't "see" E-fields, but you CAN see time-varying E-fields - that's precisely what light is! We'll talk about this more later .

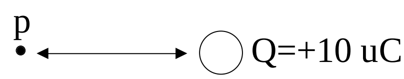
If you have many charges, finding E is little more than an exercise in adding vectors: you just imagine putting a test charge q down, and then, as before:

E = **F**_net(on q) / q.

Finding **E** (or equivalently **F**_net) can be a chore if there are lots of charges. But in the lab, finding **E** is fairly easy -just put a test charge there, and measure the force on it! That's it.

<u>Directions:</u> **F** = q**E** **=>** the direction of **F** is the same as **E** (if q>0)
If e.g. **E** points west (and if q is positive) **F** is also west.
However, if q is negative, q**E** would be east (recall, multiplying a vector by a negative number flips the DIRECTION of the vector!)

<u>*Example:*</u> Put a +10 uC charge at the origin. Look at a point p some distance to the LEFT of the origin.
What is the direction of E there?
What is the direction of force on a + test charge there?
What is the direction of force on a - test charge there?



Answers: Remember, E fields point away from + charges.
At point p, *E points left.*
The direction of force on a "+" test charge is the same as **E**, left.
(Like charges should repel. Q and "q" are both "+", they repel, it all makes sense)
The direction of **F** on a "-" test charge opposes **E,** i.e. to the right
(Makes sense. The q and Q are now opposite, so they attract)

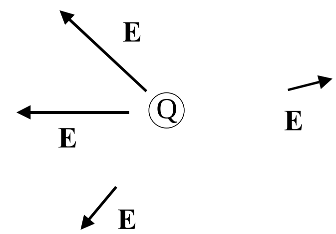
If the charge "Q" had been -10 uC in the previous example, ALL of the answers would be reversed:
In that case the E field at p would point right, TOWARDS the "-" charge.
The force on a "+" test charge would be the same directon as **E**
(that's always true!), i.e. to the right. (Makes sense. The test charge is now different from Q, so the force is attractive.)

The force on a "-" test charge opposes **E** (always true!) In this case, left.
(Again, makes sense. The test charge is now the same as Q here, both are "-", so the force between them is repulsive. At point p, that means it points left, away from Q.)

This is all a bit confusing to write, but it's really not that bad if you just think about the above examples a little.

Example: *Suppose we put a positive charge at the origin. Draw the E field at a few (randomly chosen) points.*

The E field points radially outward (radially means directly away from the Q, like a "radius" of a circle.)
The farther away you get, the weaker it is (because force, and thus E, drops off like 1/r^2)



This kind of drawing is a little tedious, and picking points at random doesn't seem like the best way of drawing an E field. But alas, E is defined *everywhere*, and it's a vector, so you really cannot draw the field in any easy way. If you drew a vector at every point, the vectors would overlap and the drawing would get very confusing.

Electric Field Lines

There is a neat pictorial trick that people use to try to "visualize" E-fields. Instead of drawing vectors at points, we draw ***field lines***. It is important to understand the field lines are a *representation* - they must be interpreted. They help us visualize the field, but it is essential to understand what these lines represent.

There are several rules of field lines that help us draw and interpret this representation:

1. At any point on the line, the ***TANGENT*** to the line at that point tells us the direction of the E-field. (The lines have arrowheads on them to eliminate any ambiguity in direction).

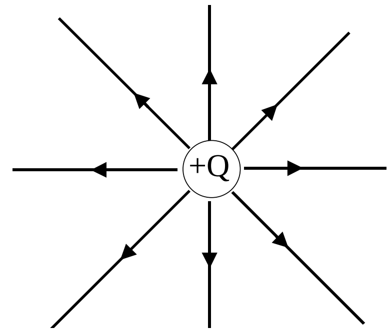
2. The more lines you have (the *denser* they are), the *stronger* the E field. (If you double the charge, you double the "density of lines".)

3. Lines never cross (if they did, the direction of the E-field would be ambiguous at the crossing. But there must always be a unique force on any charge. For a static E-field, a charge does not come to a certain point and sometimes feel a force one way and sometimes feel a force another way)

4. Field lines start and end at charges. This statement is always true. The field "comes" from somewhere. However, many times we show a diagram of just a portion of the field, so you may not always see where the field is starting and ending.

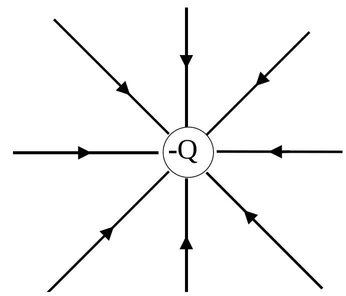
Here are several examples.

Example 1: *A single "+" charge.*
The location of the arrowheads is not significant.
The arrows all point away from the + charge
(E fields go AWAY from positive charges)
The lines are all radially outward.
Notice that the lines are less dense further away from the charge, which tells you the E field is weaker out there.
(this is just Coulomb's law, E drops like 1/r^2!)



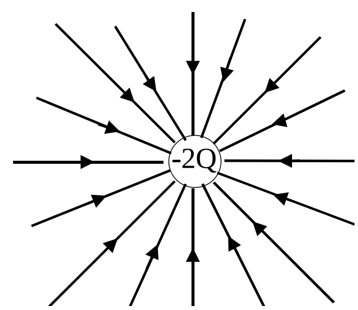
Example 2: *A single "-" charge.*

The arrows all point t*owards* the - charge
(E fields go towards negative charges)
The lines are all radially inward.
Notice that the lines are again less dense further away, (this is just Coulomb's law, E drops like 1/r^2!)



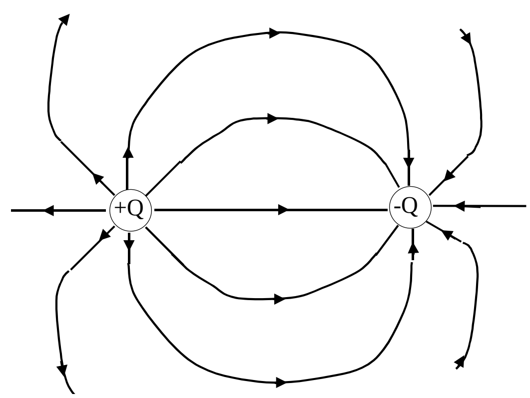
Example 3: A single charge of -2.

Just like the last one, but the density of lines is twice as much, because the charge is twice as big.



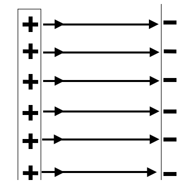
Example 4: A dipole
(that means "two poles" : + on left, - on right)

The lines are curved now. At any point in space, the E field is given by the *tangent* to the line. (Giancoli Fig 16-29a is better drawn version of this same fig!)
As always, the arrows tell you the direction of E, which is the same as the direction that a *positive* test charge would move, if released at that point. (A negative test charge would go the opposite way!)



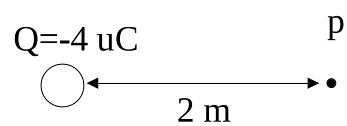
Look at the picture and convince yourself that the directions make sense: think about which way a "+" test charge would want to go given those two charges +Q and -Q.

Example 5: *Field lines between two infinite "lines" of uniform charge.*



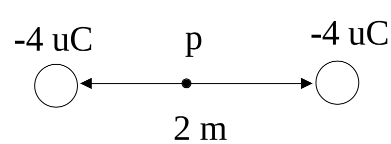
The lines are uniform density, the E field here is the same size and direction everywhere! Wherever a test charge may be, it is pushed to the right with the same force anywhere... (It may not be totally obvious to you that the E field is *uniform* here, but it is. This is a case where working it out from Coulomb's law is hard, because there are an infinite number of charges in the story! There are fancier tricks that can be used to deduce E fields which we won't cover in this course, that make this a surprsingly easy problem to solve quantitatively)

Example:
A -4 uC charge is at the origin, as shown. What is the E field at a point 2 m to the right of the origin?



Answer:
The force on a test charge q at point p would be k Q q/r^2
(that's just Coulomb's law)
E=F/q, so
E = k Q/r^2 = (9E9 N m^2/C^2)(-4E-6 C)/(2 m)^2 = 9E3 N/C
The direction of the E field is towards the ("-") charge; to the left.
(A "+" test charge at p would move to the left, opposites attract)

Example: Two -4 uC charges are set up as shown. What is the E field right in the middle, between them?



Answer: The force on a "+" test charge at point p would be to the RIGHT from the right-hand charge, and to the LEFT from the left hand charge (opposites attract, the test charge I'm imagining is +.) Since I picked p in the middle, the forces exactly *cancel*, F_net=0, so E=0 at p. If I shift a little away from the center, however, the forces won't exactly cancel, and there will be a nonzero E field. To find it at any point "off-axis" (anywhere besides on the line shown between the charges) I'd have to add the two forces as *vectors*, a bit of a pain.

In the end, the field lines look exactly like the ones shown in Giancoli 16-29b (except my 2 charges are "-", so the arrows would all be reversed.) Notice in that figure there is zero density of lines right in the middle, which corresponds to the answer E=0 we just got.

Think about what you get a little away from the center, and see if the field lines in Giancoli's picture make physical sense to you.

Field lines are very useful - they're a pictorial way to "visualize" the force that any test charge would feel anywhere in space. But, if you want to be quantitative, you usually need to compute (or be told) the value of E fields, it's hard to get numerical values from field line graphs.

E-Fields and Metals

I said **E** is defined everywhere in space. What is the electric field **E** *inside* a chunk of metal? I claim, in steady state, the answer must be zero! Why? Because metals are filled with free electrons that can move around (they conduct) - so if E was NOT zero at some point inside, then the force on an electron there would be nonzero - the electron would start to move. They would continue to move, building up a "counter" E field, a canceling field, until finally everything settles down, nothing moves, E=0 throughout the metal (so F=0) No more motion! (Steady state MEANS everything has settled down, no charges are moving, so the feel no net force on them any more)

The same argument tells you that, in steady state, if there's a nonzero E field outside of a chunk of metal, it will always be perpendicular to the surface right at the edge. If it wasn't perpendicular, there would be a component of E parallel to the surface, and electrons right at the surface would then flow (surface currents), until that piece of the E field got cancelled out. (Look back at the figure I drew in Example 5. If those charges resided on metal plates, the E field lines would have to be perpendicular to the plates - just as I drew it)

Comments

There are tons of practical consequences of the above simple statements. Just one example: if you make a metal box and put it in a region of large E-field, the electrons in the metal quickly (essentially instantly) rearrange on the surface to make the E-field equal zero ***inside*** the box. If you then hollow out the box, it makes no difference; the E-field is still zero inside. In a lightning storm (large, potentially fatal E-fields all around you), a relatively safe place to be is inside a metal car, because the E field inside the car is zero. (It would be better if the car was *entirely* metal) If you are in a fancy fiberglass-body car, too bad - fiberglass does not conduct, it is not a metal, so the above arguments fail to hold. At least you'll stay dry...

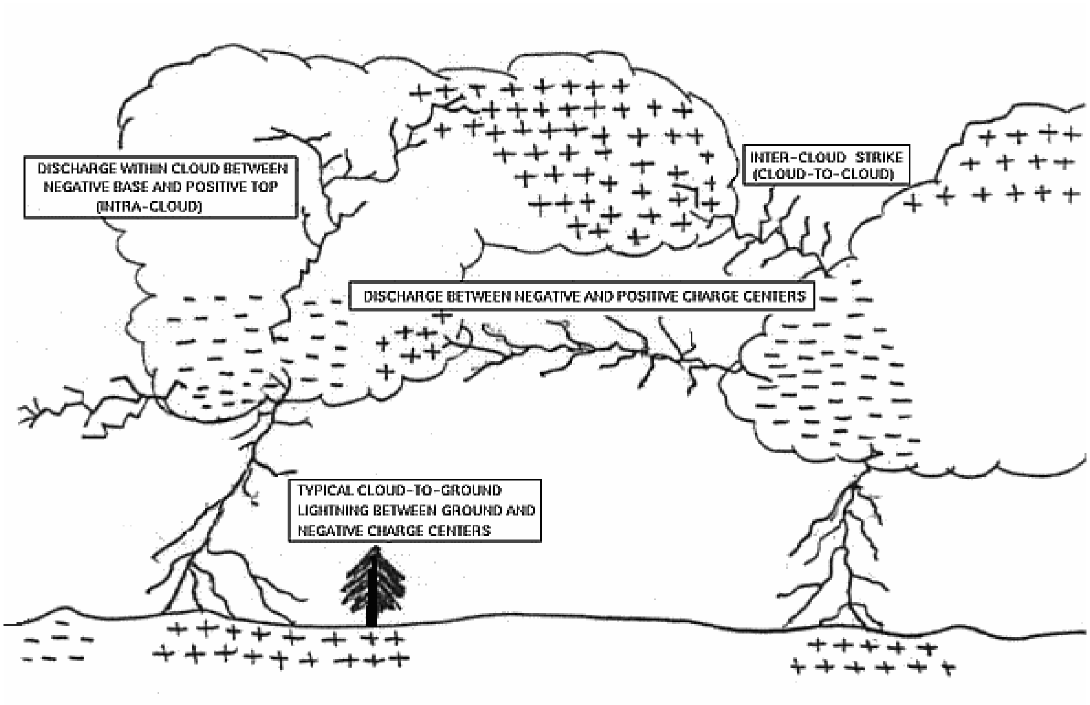
Giancoli ends this chapter with a cool section on electric forces in DNA: biophysics is an incredibly interesting and rapidly growing field of study. The interplay of basic laws of physics in biological systems allows us to understand, and control, living systems in ways that were unimaginable just a few years ago. Electric fields make forces on charges, shaping and controlling E field allows you to manipulate charged objects (which ultimately means just about everything!)

Aside: Some Lightning Physics:

(http://thunder.msfc.nasa.gov/primer)

The story of lightning is pretty complicated, but here's a summary of some of the basic ideas of "ordinary" lightning: In a thundercloud, "+" charges tend to get swept up to towards the top, "-" are down low (the *mechanism* for this charge separation is not yet fully understood - nice to know that the physics of something as elemental as lightning still has some scientific mystery associated with it!) and the large electric fields near those charges begin to break down the air molecules, forming paths of "ionized" (broken down) air, which is a much better conductor of electricity than normal air. This process is called forming a "stepped leader", because it tends to occur in steps or jumps of 50-100 m. The path is pretty random, although it tends to work its way down towards the ground (which is inductively charged the opposite way). Meanwhile "positive streamers" (which are like the stepped leaders only they're working their way ***up*** from the ground, usually from tall or sharp objects - we'll see why later) are forming near the ground. When a "-" stepped leader path meets a "+" streamer path, there's now a *continuous* path of good conductivity between ground and cloud, and BOOM there's a lightning bolt.

The "flash" arises from the high temperatures associated with the energy being dumped into the air from all those charges passing by, and also from electrons getting re-attached to molecules after they're stripped off. The flash usually works its way ***up*** from the point where the leaders joined the streamers, up towards the cloud (so in this sense, lightning goes ***up*** from the ground to the cloud!) There can be one flash, or a couple of quick strokes through one path (typically 4, each lasting about 30 microseconds). The "boom" of thunder is the shock wave of the air as this superheated column (20,000 C, hotter than the surface of the sun!) expands and then contracts again. Since sound travels so much slower than light, you hear the boom later than you see the flash. If you count, you can estimate how far away the bolt was - sound goes about 1 mile in 5 secs...


DISCHARGE WITHIN CLOUD BETWEEN NEGATIVE BASE AND POSITIVE TOP (INTRA-CLOUD)
INTER-CLOUD STRIKE (CLOUD-TO-CLOUD)
DISCHARGE BETWEEN NEGATIVE AND POSITIVE CHARGE CENTERS
TYPICAL CLOUD-TO-GROUND LIGHTNING BETWEEN GROUND AND NEGATIVE CHARGE CENTERS